GOVERNMENT COLLEGE OF ENGINEERING & CERAMIC TECHNOLOGY
AN AUTONOMOUS INSTITUTE
AFFILIATED TO MAKAUT (FORMELY KNOWN AS WBUT)

Theory / B. Tech. / IT / SEM - IV / Code – BS(IT) 408 / 2017-18

Paper Name: Discrete Mathematics

Full Marks: 75 **Time Allotted: 3 hours**

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
[MCQ Type Questions][Compulsory]

1. Choose the correct alternative of the following questions. Answer all questions. **10 x 1 = 10**

i) The proposition $p\wedge(\sim p \vee q)$ is

a) a tautology b) logically equivalent to $p \vee q$

c) logically equivalent to $p\wedge q$ d) none of these

ii) Negation of $\forall x\, P(x)$ is

a) $\forall x\, P(x)$ b) $\sim(\exists x)\, P(x)$

c) $(\exists x)\, (\sim P(x))$ d) none of these

iii) The remainder when -69 is divided by 8 is

a) 1 b) 3

c) 5 d) 6

iv) A POSET S is a lattice if every pair of elements of it has

a) g.l.b. and l.u.b. is S b) l.u.b. in S

c) g.l.b. in S d) maximal and minimal elements in S

v) The solution of the recurrence relation $a_n = 2\,a_{n-1}$ with initial condition $a_0 = 1$ is

a) $1 - 2^n$ b) 2^n

c) 2^{n-1} d) $2^n - 1$

vi) The clique number of a bipartite graph $K_{m,n}$ is

a) 1 b) 2

c) 3 d) 4

vii) A connected planar graph has 5 edges and 2 regions. The number of vertices of the graph is

a) 4 b) 2

c) 5 d) 3

viii) If G is a tree with 50 vertices then the chromatic number of G is

a) 50 b) 100

c) 150 d) none of these

ix) Number of ways of a tree with 4 vertices can be coloured with at most 3 colours is

a) 12 b) 24

c) 36 d) 48

x) For a perfect matching the corresponding graph in a matching problem should be

a) bipartite graph b) a cycle having even number of vertices

c) a complete graph d) none of these

GROUP – B

[Short Answer Type Questions]

Answer any *four* of the following

4 x 5 = 20

2. Verify the validity of the following argument: All integers are rational numbers. Some integers are powers of 2. Therefore some rational numbers are power of 2.

3. Determine the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 32$, where x_1, x_2, $x_3 > 0$ and $0 < x_4 \leq 25$.

4. Show that in a Lattice $(L, \wedge, \vee)$, $a \wedge c \leq b \wedge c$ and $a \vee c \leq b \vee c$ if $a \leq b$.

5. If the gcd(a,b)=1, then prove that gcd(a+b,a-b)=1 or 2.

6. State and prove Euler's theorem for a connected planar graph.

7. Prove that chromatic polynomial of cycle C_n ($n \geq 3$) with n vertices is $(x-1)^n + (-1)^n(x-1)$.

Group – C

[Long Answer Type Questions]

Answer any *three* of the following

3 x 15 = 45

8. a) Find the number of different non-equivalent statement formula containing n statement letters.

b) f is continuous if g is bounded or h is linear. g is bounded and h is integrable if and only if h is bounded or f is not continuous. If g is bounded then h is unbounded. If g is unbounded or h is not integrable, then h is linear and f is not continuous. --- Check the consistency of the following set of assumptions.

c) For a statement formula $\sim((p \rightarrow q) \leftrightarrow \sim(p \wedge \sim q))$ find an equivalent statement formula in full CNF. [4+7+4]

9. a) Find the remainder when 1! + 2! + 3! + ……….. + 100! is divided by 30.

b) Find the chromatic polynomial of a complete graph K_n with n vertices.

c) Show that $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is a tautology

d) Find the minimum number of students in a class to be sure that 6 of them are born in the same month. [3+5+4+3]

10. a) A doctor gives a prescription of 20 tablets to a patient with the instruction to take at least one tablet per day for 15 days. Show that there is a period of consecutive days during which the patient takes a total of 9 tablets.

b) Use generating function to solve the recurrence relation $a_n - 9a_{n-1} + 20a_{n-2} = 0$, $n \geq 2$, $a_0 = -3$ and $a_1 = -10$.

c) Show that the square of an odd integer is of the form (8k+1), where k is a non-negative integer.

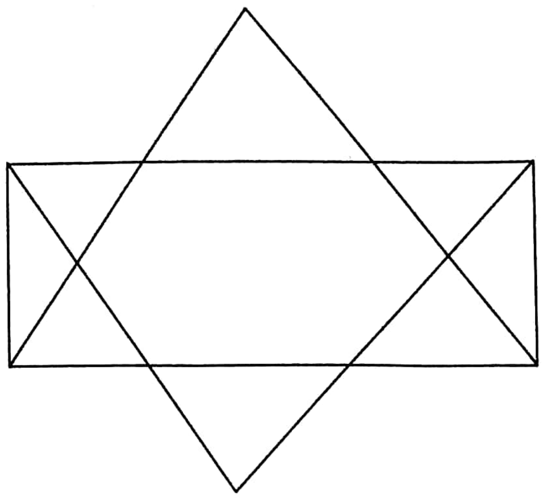
d) Prove that every distributive lattice is modular. [4+5+3+3]

11. a) If every region of a planar graph with n vertices and e edges embedded in a plane is bounded by k edges then show that $e = \frac{k(n-2)}{k-2}$.

b) Show that every simple connected planar graph G with less than 12 vertices must have a vertex of degree less than equal to 4.

c) Give an example of a graph which is self dual.

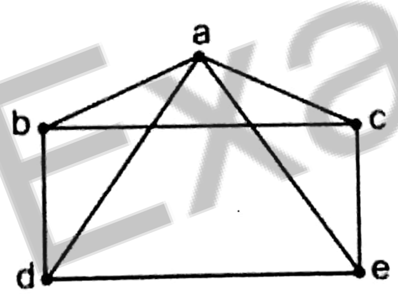
d) Find the dual of the following graph. [4+4+2+5]



12. a) Prove that for any graph G with n vertices $\frac{n}{\beta(G)} \leq \chi(G) \leq n - \beta(G) + 1$, where $\chi(G), \beta(G)$ denotes the chromatic number and independence number of G.

b) Show that there always exists a perfect matching for a k-regular bipartite graph.

c) Use decomposition rule to find the chromatic polynomial of the given graph and hence find the chromatic number of this graph. [5+5+5]



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